WRITTEN CALCULATION POLICY

Written calculation

Our calculation policy supports children's development through the concrete, pictorial and abstract (CPA) stages of mathematical learning. It provides teachers with a clear framework for progression in maths across the whole primary age range.

The expected methods, models and apparatus which are suitable at each level of a child's mathematical understanding are outlined for each of the 4 operations.

By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave our schools they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

CONCRETE PICTORIAL ABSTRACT + 2+1=3

Progression towards a standard written method of calculation

INTRODUCTION

The PA maths programme provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies. Up to the age of 7 (Year 2) informal written recording should take place regularly and is an important part of learning and understanding. More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.

REASONS FOR USING WRITTEN METHODS

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation

STANDARD VOCABULARY FOR EACH OPERATION

+	−
Get some more, real story, maths story, same value different appearance, tens, units, hundreds, thousands, place value, digit, value, combine, sum, total, add, addition, more, plus, increase, sum, total, altogether, score, double, near double, how many more to make…?, equals, sign, regroup, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse, fair swap	Get ready to take away, real story, maths story subtract, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is.. than…?, how much more/less is…?, equals, sign, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse, regroup, fair swap
x	**÷**
Lots of, groups of, I love that number- how many times? times, product, multiply, multiplied by, multiple of, once, twice, three times, four times, five times,… ten times, repeated addition, array, row, column, double, regroup, fair swap, inverse	Halve, share, share equally, one each, two each, three each…, group in pairs, threes… tens, equal groups of, divide, divided by, divided into, divisible by, remainder, factor, quotient, inverse, regroup, fair swap, inverse

We have developed a consistent approach to the teaching of written calculation methods. This will establish continuity and progression throughout the school.

Mental methods will be established. These will be based on a solid understanding of place value in number and will include the following:

i. Remembering number facts and recalling them without hesitation. *e.g. pairs of numbers which make 10*

   Doubles and halves to 20

ii. Using known facts to calculate unknown facts. *e.g. 6 + 6 = 12 therefore 6 + 7 = 13*

   24 + 10 = 34 therefore 24 + 9 = 33

iii. Understanding and using relationships between addition & subtraction to find answers and check results.

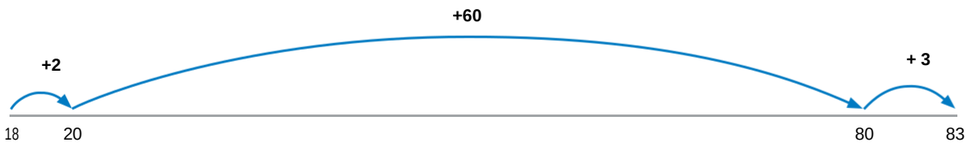
   e.g. 14 + 6 = 20 therefore 20 – 6 = 14

iv. Having a repertoire of mental strategies to solve calculations *e.g. doubles/near doubles*

   Bridging 10/bridging 20

   Adding 9 by + 10 and – 1

v. Making use of informal jottings such as blank number lines to assist in calculations with larger numbers *e.g. 83 – 18 = 65.*



vi. Solving one-step word problems (either mentally or with jottings) by identifying which operation to use, drawing upon their knowledge of number bonds and explaining their reasoning

vii. Beginning to present calculations in a horizontal format and explain mental steps using numbers, symbols or words

viii. Learning to estimate/approximate first e.g. 29 + 30 (round up to nearest 10, the answer will be near to 60).

Place value will be taught mentally first from Reception class where number tracks are used, progressing to number lines (to 10 or 20 as appropriate) in Years 1 and 2.

The empty number line will then be introduced to aid calculations.

Subtraction will be taught by counting on and counting back depending on the numbers.

Numbers such as 10, 100, 1000 will be called Landmark Numbers.

WHEN ARE CHILDREN READY FOR WRITTEN CALCULATIONS?

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division

- Do they know the 2, 3, 4, 5 and 10 time table
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

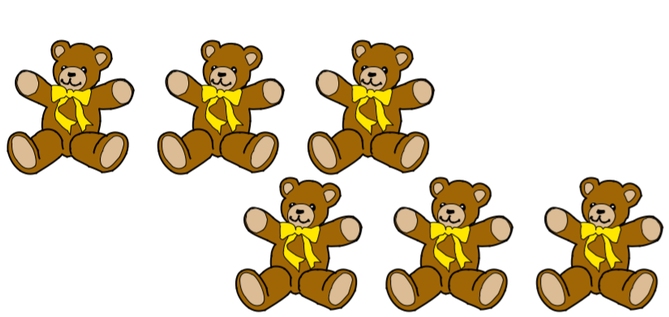
Addition

EYFS

Recognise numbers 0 to 10.

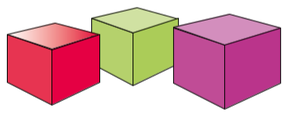
0 1 2 3 4 5 6 7 8 9 10 7 0

Count reliably up to 10 everyday objects.

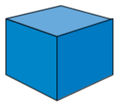


1,2,3,4,5,6…
there are 6 teddies

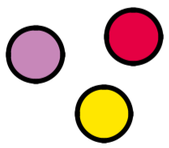
Find one more than a number.



One more than three is four.



Begin to relate addition to combing two groups of objects.



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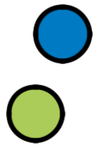
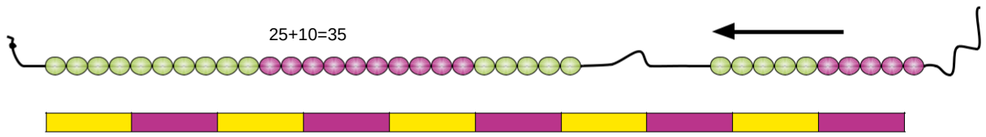


makes 5

EYFS/Year 1

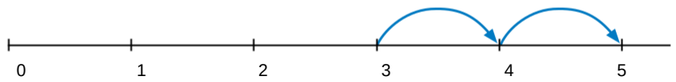
Count in ones and tens.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



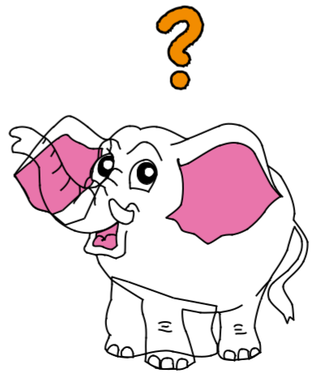
Count along a number line to add numbers together.

3+2=5



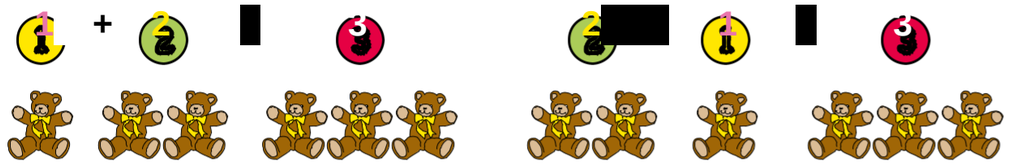
Begin to use the + and = signs to record mental calculations in a number sentence.

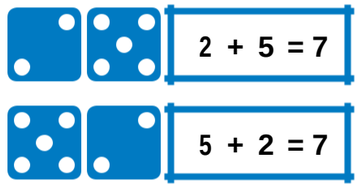
6+4=10



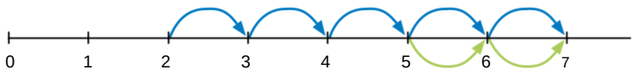
Year 1

Know that addition can be done in any order.



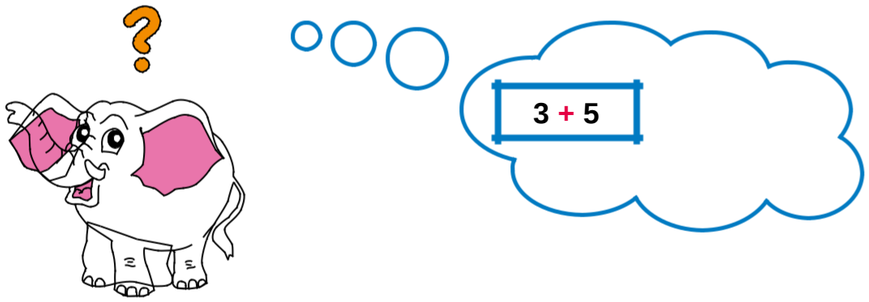


2 count on 5

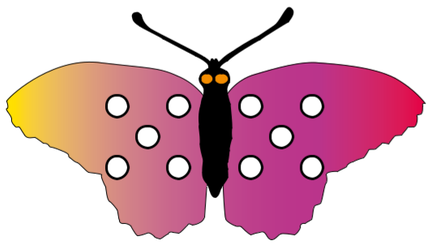


5 count on 2

Put the biggest number first and count on.



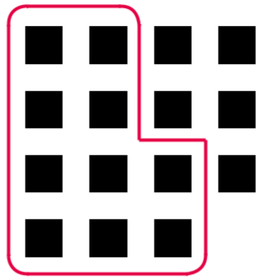
Know doubles of numbers.



5+5=10

Year 2

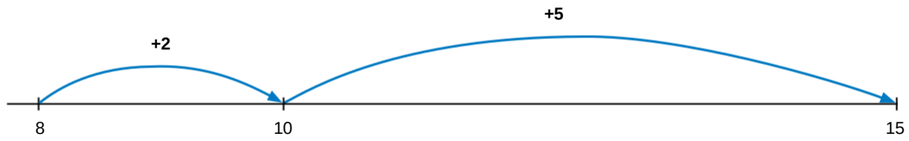
Add two single digit numbers that bridge 10.



8+8=15

Encourage children to partition numbers to use bonds to ten.

Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.

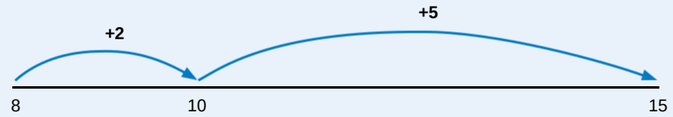
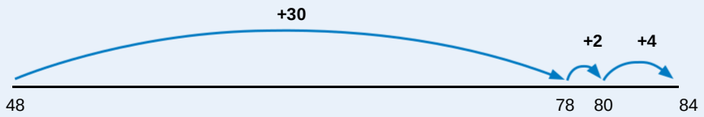
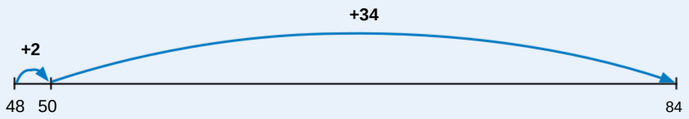
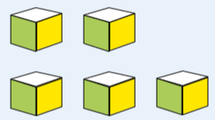
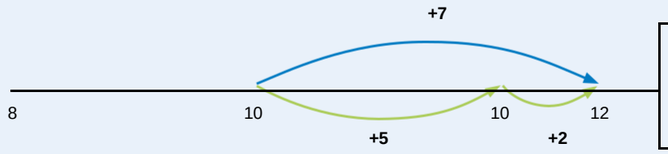
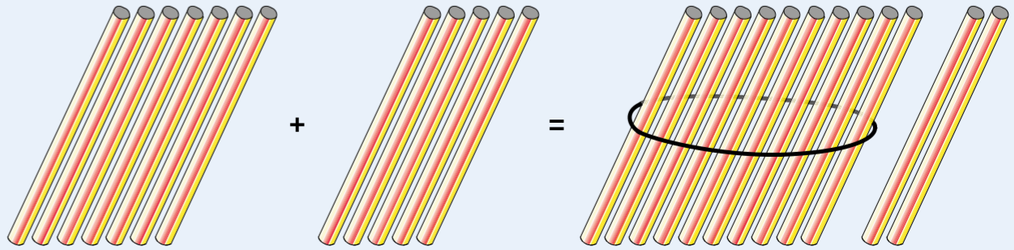


The empty number line helps to record the steps on the way to calculating the total.

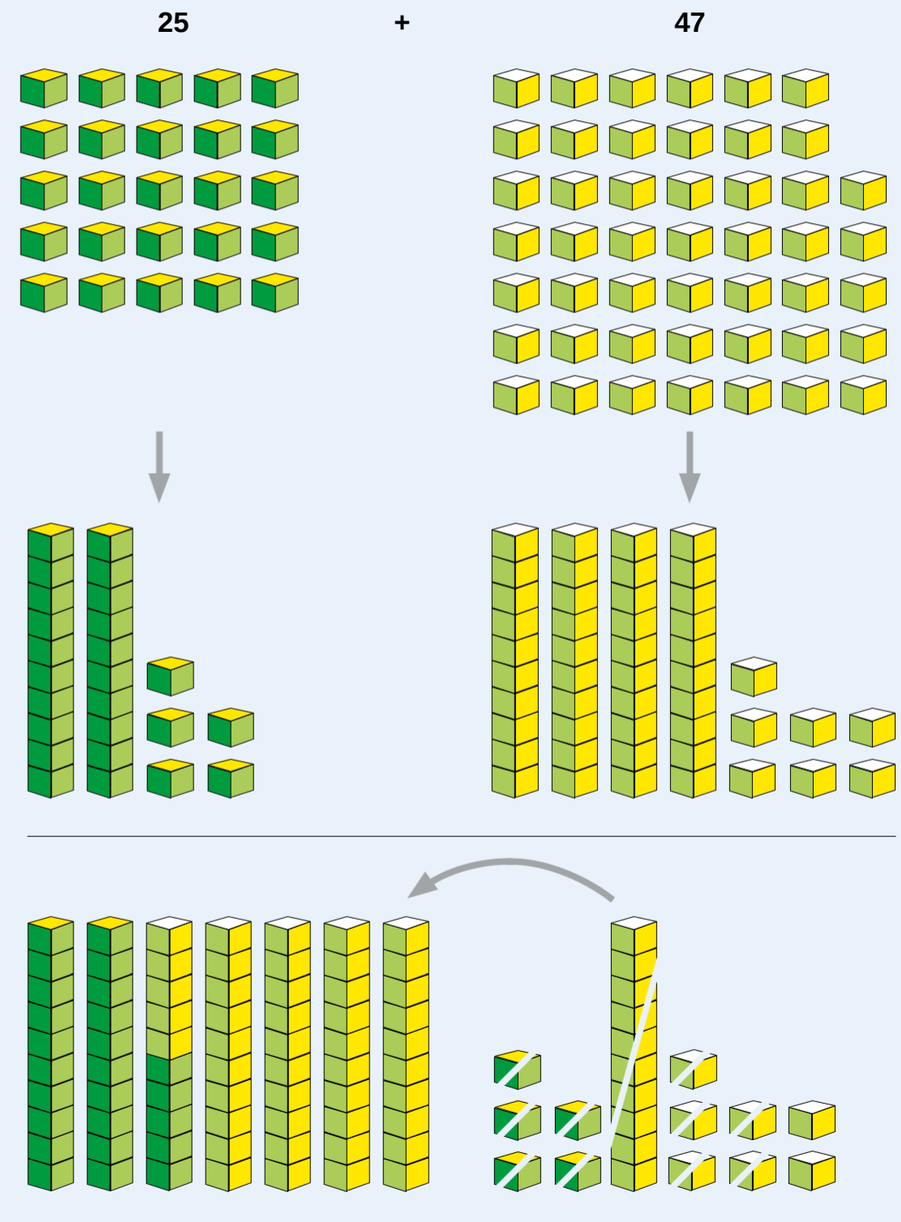
CONCRETE PICTORIAL ABSTRACT 2+1=3

Written methods for addition

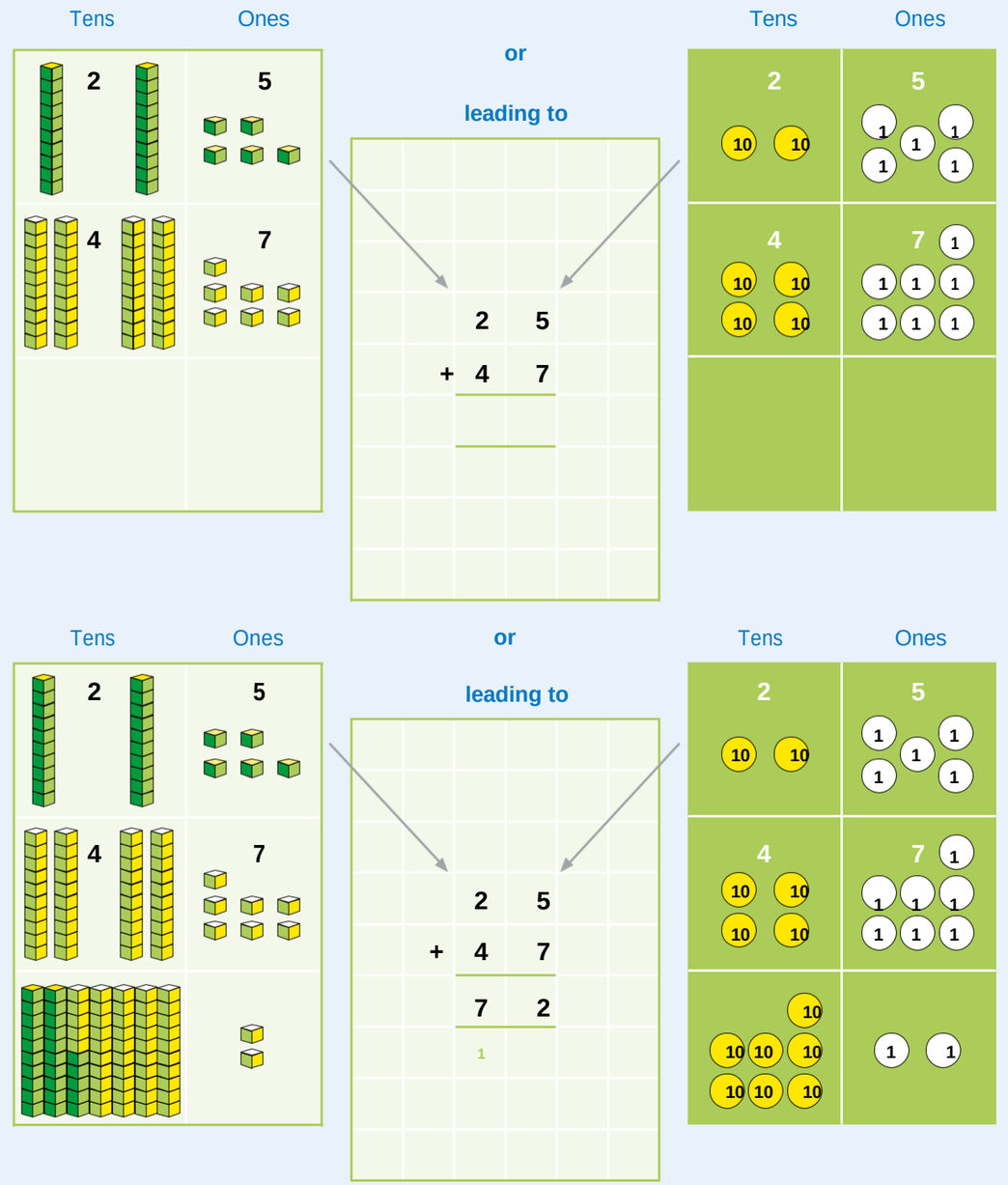
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for each of the four operations (addition, subtraction, multiplication and division) which they know they can rely on when mental methods are not appropriate.

Mental calculation strategies	Stage 1: The empty number line	CONCRETE APPROACHES
• Counting on in ones and then 10, 5 and 2 using a number line and without • Know by heart all pairs of numbers with a total of 10 • Addition facts for all pairs of numbers with a total of up to at least 5 and corresponding subtraction facts • Doubles of numbers to at least 5 • Identify near doubles, using doubles already known (5 + 6) • Add 9 to single-digit numbers by adding 10 and then subtracting 1 • Begin to bridge 10 when adding a single-digit number	After much practical work to build an understanding steps in addition can be recorded in single step jumps on a number line. This develops on to bridging through a multiple of 10. 8+7=15  48+36=84  or: 	 5 12 7 Counting on with a bead bar/number line.  **Also:** Bead bar and number line (showing 10s) encourages use of number bonds and place value for added effeciency. Counting on with straws. 

CONCRETE PICTORIAL ABSTRACT 2+1=3

Mental calculation strategies	Stage 2: Partitioning	CONCRETE APPROACHES
• Counting on in 3s and 4s • Know by heart all pairs of numbers with a total of 20 • Know by heart all pairs of multiples of ten with a total of 100 • Addition facts for all pairs of numbers with a total of up to at least 10 and corresponding subtraction facts • Doubles of numbers to at least 10 and multiples of 10 to 100 • Identify near doubles, using doubles already known (40 + 41) • Add 9 or 11 to by adding 10 and then adding or subtracting 1 • Bridge 10 when adding a single-digit number	Record steps in addition using partitioning: 47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 Partitioned numbers are then written under one another:	

CONCRETE PICTORIAL ABSTRACT 2+1=3

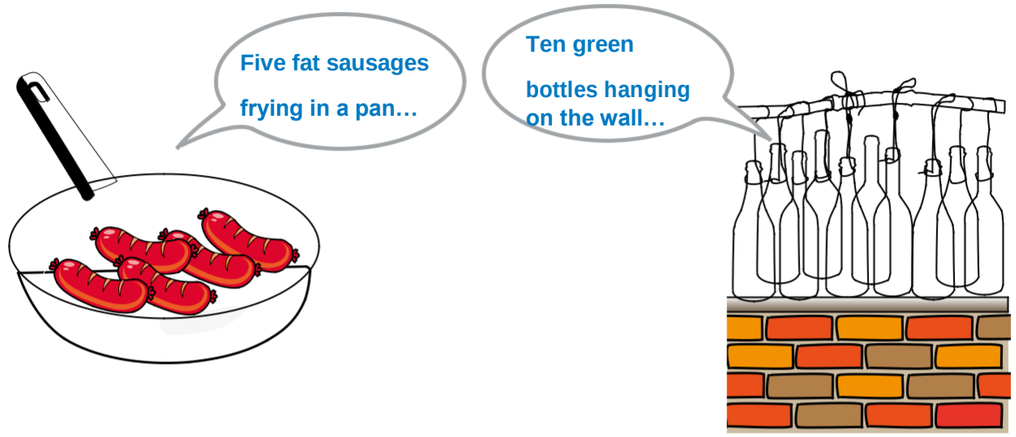
Mental calculation strategies	Stage 3: Expanded method in columns	CONCRETE APPROACHES
• Know all addition facts for numbers to 20 • Derive quickly all pairs of multiples of 5 with a total of 100 • Partition into tens and unit/ones, then recombine • Doubles of all whole numbers to at least 20 • Doubles of multiples of 5 to 100 • Doubles of multiples of 50 to 500 • Identify near doubles, using doubles already known (80 + 79) • Add a near multiple of 10 to or from a 2-digit number by adding 10 and adjusting • Bridge through a multiple of 10 and adjust • Use known number facts and place value to add mentally	Write the numbers in columns. Adding the tens first. Adding the ones first. Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.	

Mental calculation strategies	Stage 4: Column method
• Partition into tens and units/ones, adding tens first • Identify near doubles using doubles already known (150 + 160) • Add the nearest multiple of 10, then adjust • Add 3 or 4 small numbers, finding pairs totalling 10, 9 or 11 • Add three 2-digit multiples of 10 • Use doubling starting from known facts	Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Written methods for subtraction

EYFS

Begin to count backwards in familiar contexts such as number rhymes or stories.



EYFS/YEAR 1

Continue the count back in 1s from any given number.

10, 9, 8, 7…

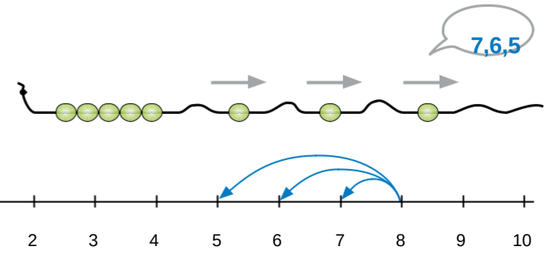
Begin to relate subtraction to taking away'.

Find one less than a number up to 10.

1 less than 8 is? 7

2 less than 8 is? 7, 6

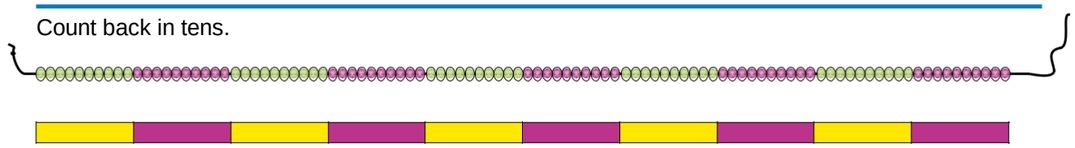
3 less than 8 is? 7,6,5



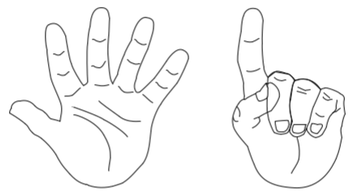
Count back one, two or three.

YEAR 1

Count back in tens.



Begin to use the – and = signs to record a number sentence using numbers up to 10 whilst still using practical resources.



6–4=2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

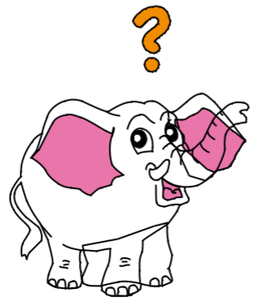
Compare 2 sets to find the numerical difference.

The difference between 8 and 5 is 3.

Year 1

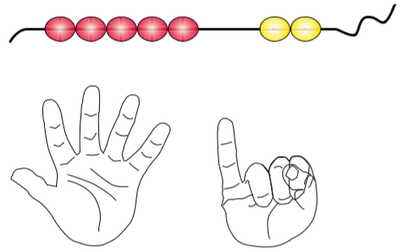
Understand the operation of subtraction and use related vocabulary.

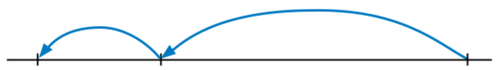
Subtract numbers when solving problems involving up to 10 objects in a range of contexts.

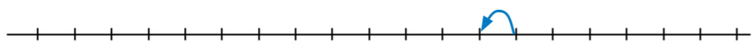


Maria had six sweets and she ate four. How many did she have left?

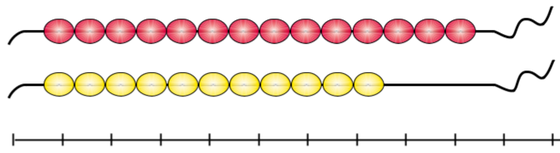
6–4=2

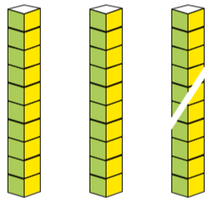


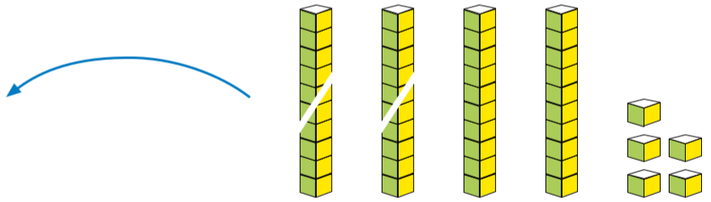


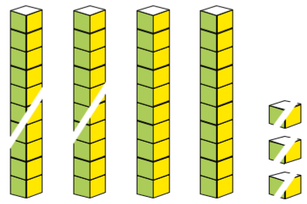


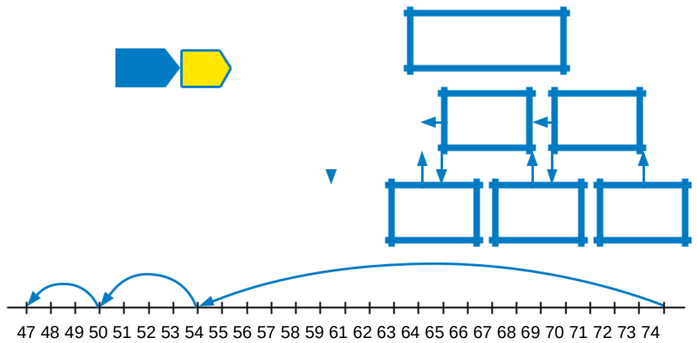
CONCRETE PICTORIAL ABSTRACT
2+1=3







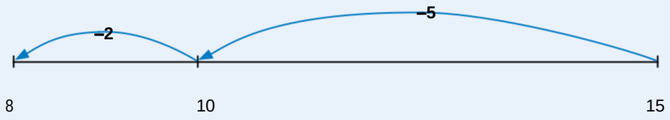
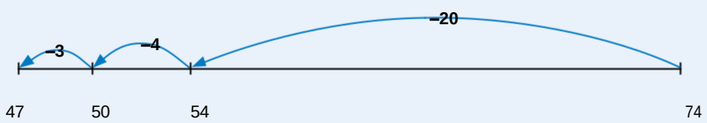
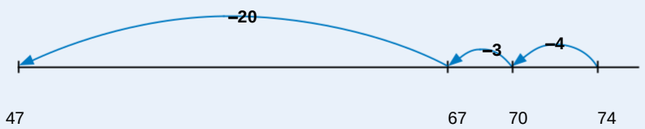
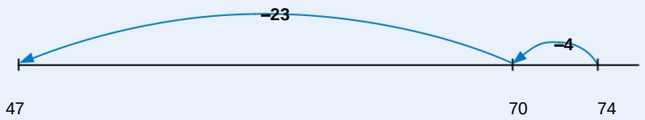
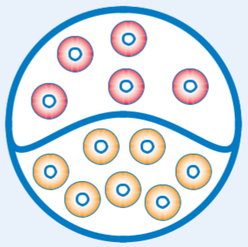




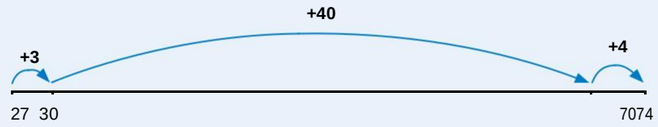
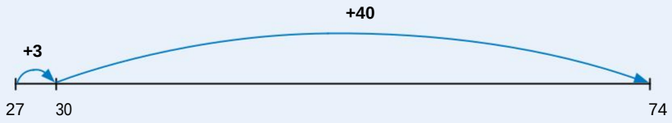
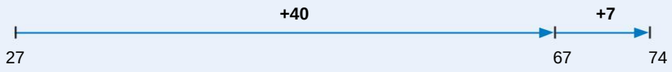
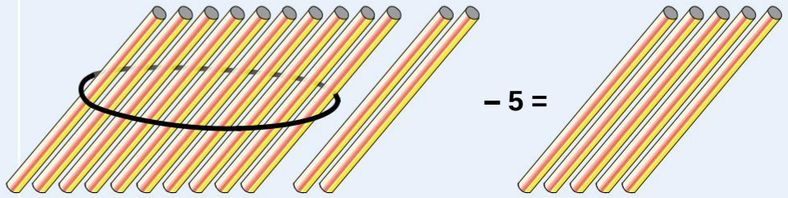
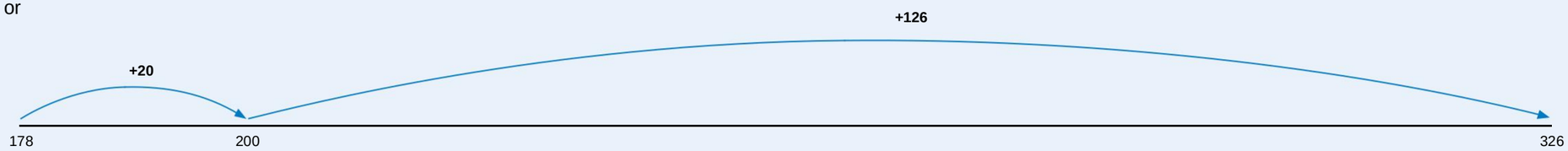
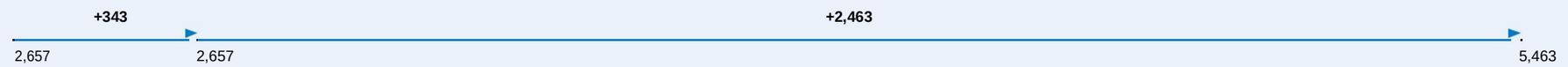
CONCRETE PICTORIAL ABSTRACT 2+1=3

Written methods for subtraction of whole numbers

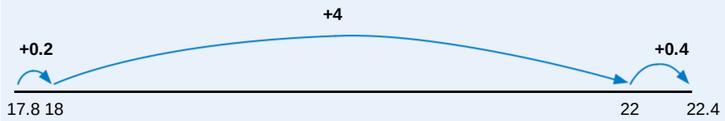
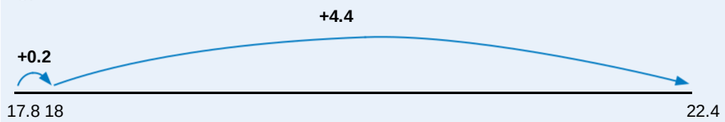
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

Mental calculation strategies	Stage 1: The empty number line	CONCRETE APPROACHES
• Counting back in ones and then 10, 5 and 2 using a number line and without • Know by heart all pairs of numbers with a total of 5 and corresponding subtraction facts • Begin to know addition facts for all pairs of numbers to 10 and corresponding subtraction facts	After much practical work to build an understanding steps in subtraction can be recorded on a number line. This develops on to bridging through a multiple of 10. 15–7=8  74 – 27 = 47 worked by counting back:  The steps may be recorded in a different order:  Or combined: 	 12–5=7 **N.B.** When this is done on a bead bar, there are links with both counting back and difference on a number line. **Comparing two sets (comparison or difference)**  **Seeing one set as partitioned**  **Seeing 12 as made up of 5 and 7** **Issue:** Helps to see the related calculations; 5 + 7 = 12, 7 + 5 = 12, 12 – 7 = 5 and 12 – 5 = 7 as all in the same diagram.

CONCRETE PICTORIAL ABSTRACT 2+1=3

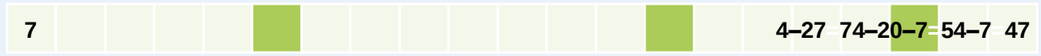
Mental calculation strategies	Stage 1: The empty number line	CONCRETE APPROACHES
• Counting back in 3s and 4s • Know addition facts for all pairs of numbers to 10 and corresponding subtraction facts • Find a small difference by counting up from the smaller number • Subtract 9/11 by adding 10 and adding/subtracting one	 or  or: 10s first 	 **Issue:** Number line helps to stop 'counting all'. **Finding the difference on a number line**  
• Counting back in small steps from a small two-digit number • Know all addition facts for each number to 20 and corresponding subtractions • Find a small difference by counting up • Subtract a near multiple of ten from a two-digit number • Use known number facts and place value to subtract mentally	 or  Leading to: 5,463 – 2,657  2,806	

CONCRETE PICTORIAL ABSTRACT 2+1=3

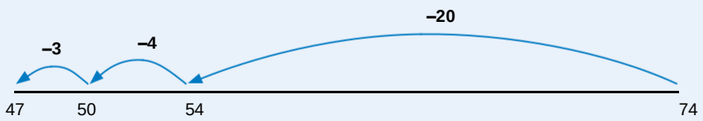
Mental calculation strategies	Stage 1: The empty number line	CONCRETE APPROACHES
• Find a difference by counting up • Count back in repeated steps of 1, 10, 100 • Subtract the nearest multiple of 10 or 100, then adjust • Use the relationship between addition and subtraction • Use known number facts and place value to subtract mentally	 or 	

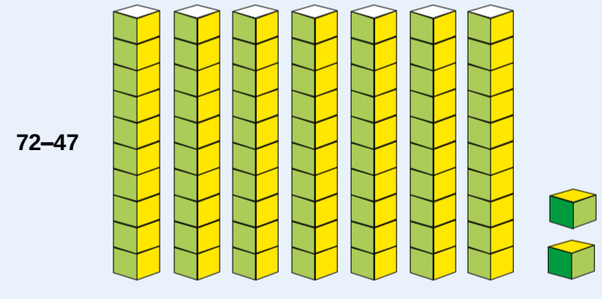
Stage 2: Partitioning

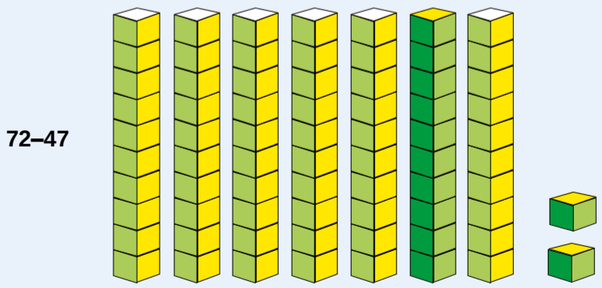
Subtraction can be recorded using partitioning:



This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.







CONCRETE PICTORIAL ABSTRACT 2+1=3

Stage 3 : Decomposition

1. **563 – 241**

	5	0	0	+	6	0	+	3	
–	2	0	0	+	4	0	+	1	
	3	0	0	+	2	0	+	2	=322

Leading to

	5	6	3
–	2	4	1
	3	2	2

2. **563 – 278**

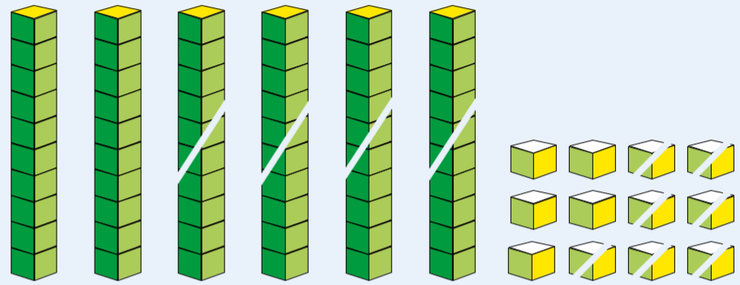
	5	0	0			+60+3	→		4	0	0		1	5	0		1	3
									5	0	0	+		6	0	+		3
–	2	0	0			+70+8	→	–	2	0	0	+		7	0	+		8
											200	+		8	0	+		5

Leading to

		4	15	1
		5	6	3
–		2	7	8
		2	8	5

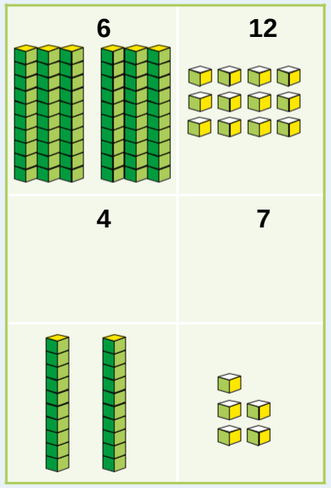
CONCRETE APPROACHES

72–47



20+5=25

Tens	Ones
6	12



or

Tens	Ones
6	12
4	7

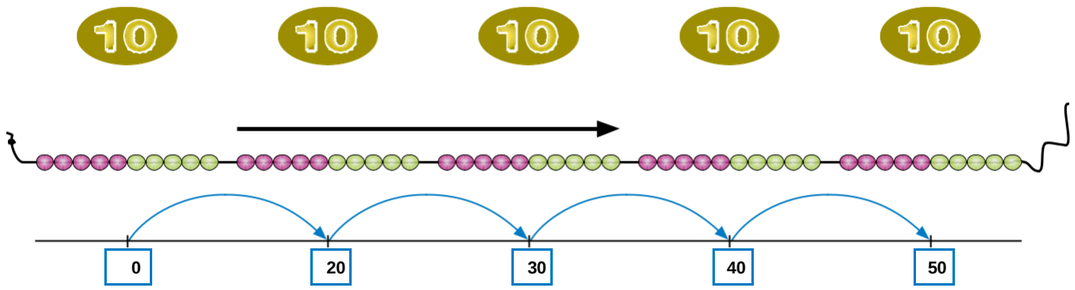
leading to

		6	1
		7	2
–		4	7
		2	5

Multiplication

EYFS

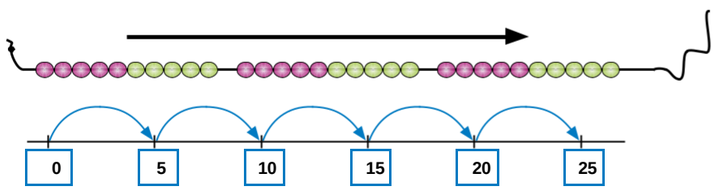
Count in tens from zero.

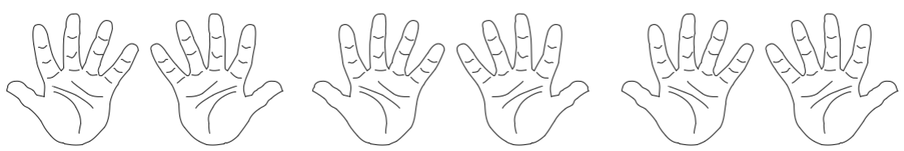


Year 1

Count in twos from zero.

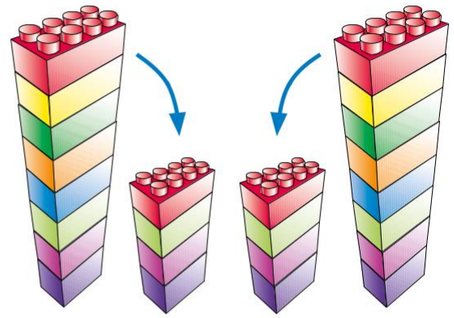
Count in 5s from zero.





Year 1

Know doubles and corresponding halves.



Half of 8 is 4.

8÷2=4

Double 4 is 8.

4 x 2 = 8

Year 2

Understand multiplication as repeated addition.

2+2+2+2=8

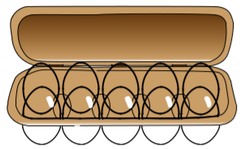
4 x 2 = 8

2 multiplied by 4

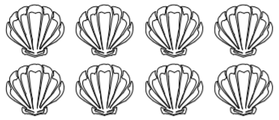
4 lots of 2

3+3+3=9

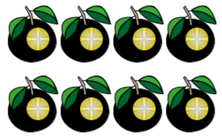
Understand multiplication as an array.



5 x 2 = 10



2 x 4 = 8



4 x 2 = 8

2 x 4 = 8

2 x 5 = 10

2 x 4

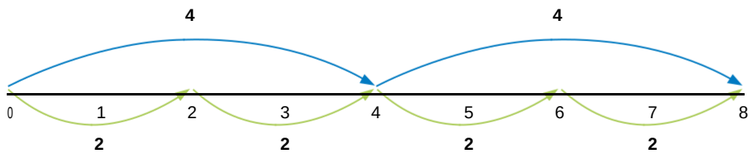
2 x 4 = 8

4 x 2 = 8

Understand how to represent arrays on a number line.

Pattern work on a 100 square helps children begin to recognise multiples and rules of divisibility.

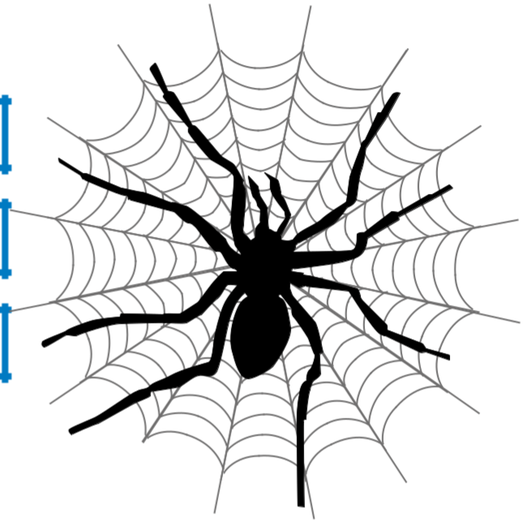
2 hops of 4



4 hops of 2

Know by heart facts for the 2, 5 and 10 multiplication tables.

x 5



2 x 5 = 10

4 x 5 = 20

10 x 5 = 50

6 x 5 = 30

3 x 5 = 15

8 x 5 = 40

5 x 5 = 25

CONCRETE PICTORIAL ABSTRACT 2+1=3

Written methods for multiplication

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

Mental calculation strategies

Derive quickly:

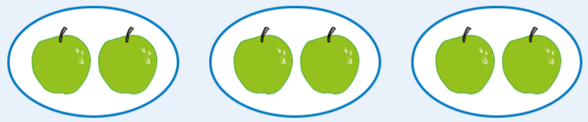
- Doubles of numbers to at least 5
- Doubles of numbers to 10 and multiples of 10
- Doubles of whole numbers to 20, multiples of 5 to 100 and 50 to 500
- Identifying near doubles from doubles already known

Know by heart:

- Multiplication facts for 2, 5 and 10 times tables
- Multiplication facts for 2, 3, 4, 5 and 10 times tables
- To multiply by 10/100, shift the digits one/two places to the left

Stage 1:: Mental multiplication using partitioning

Groups of: 2 x 3



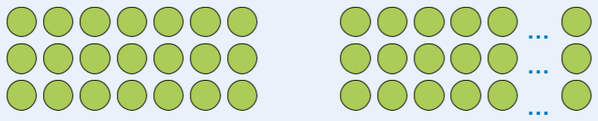
Repeated addition: **2 x 3 = 2 + 2 + 2 = 6**

Partitioning: **43 x 6**

		43			
40	+	3			
			x		6
240	+	18	=		258

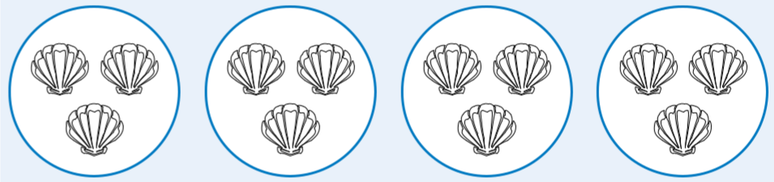
Also record mental multiplication using partitioning.

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:



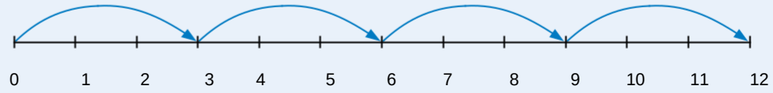
CONCRETE APPROACHES

Lots of the 'same thing'.

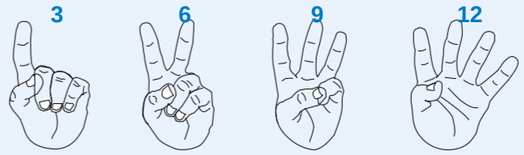


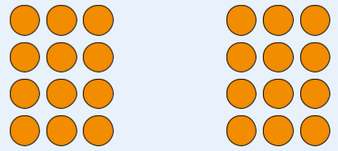
Bead bar

Number line



Fingers





3 x 4 4 x 3

CONCRETE PICTORIAL ABSTRACT 2+1=3

Mental calculation strategies	Stage 2: The grid method	CONCRETE APPROACHES

Derive quickly:

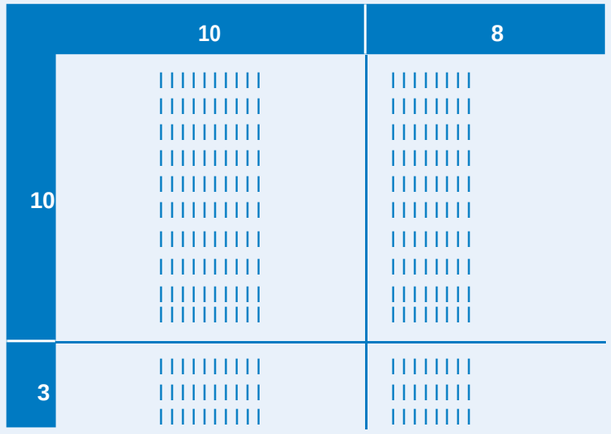
- Doubles of whole numbers to 50, multiples of 10 to 500 and 100 to 5000

Know by heart:

- All multiplication facts to 10 x 10
- To multiply by 4, double and double again
- To multiple by 5, multiple by ten and halve
- To multiply by 20, multiply by 10 and double

38×7=(30×7)+(8×7)=210+56=266

	x			7
3	0	2	1	0
	8		5	6
		2	6	6



13 x 18

Leading to grid method

Mental calculation strategies	Stage 3: Expanded short multiplication

Derive quickly:

- Use doubling starting from known facts e.g. double any two-digit number by doubling tens first
- Multiply by 25 by x 100 and finding a quarter
- Find x 16 facts by doubling x 8
- Find x 12 facts by x 10 + x 2
- Find x 17 facts by x 10 + x 7
- Find sixths by halving thirds
- Use factors e.g. 8 x 12 = 8 x 4 x 3
- Use closely related facts e.g. x 19 by x 20 and adjust
- Partition

38 x 7

3	0	x	7	=	2	1	0
8	x	7	=	5	6		
2	1	0	+	5	6	=	2	6	6
---	---	---	---	---	---	---	---	---	---

		3	8
		x	7
		5	6
+	2	1	0
	2	6	6

	10	8
10	100	80
3	30	24

CONCRETE PICTORIAL ABSTRACT 2+1=3

Mental calculation strategies

- Use relationship between multiplication and division

Stage 4: Short multiplication

		2	4
x			6
1	4	4	
	2		

		3	4	2
x				7
1	3	9	4	
	2	1		

Stage 5: Two-digit by two-digit products

56 × 27 is approximately 60 × 30 = 1800.

x	20	7	+
50	1000	350	1350
6	120	42	162
			1512
			1

	50	6	+
x	20	7	
	1000	350	1350
	120	42	162
			1512
			1

Stage 6: Three-digit by two-digit products

286 × 29 is approximately 300 × 30 = 9000.

x	20	9	+
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

			2	8	6	
			x	2	9	
				5	4	
			7	2	0	
	1		8	0	0	
			1	2		
	1		6	0		
+	4		0	0		
	8		2	9	4	
	2					

CONCRETE APPROACHES

	10	8
10	100	80
3	30	24

When?

Whether?

How?

		1	8	
	x	1	3	
		5	4	
	1	8	0	
	2	3	4	

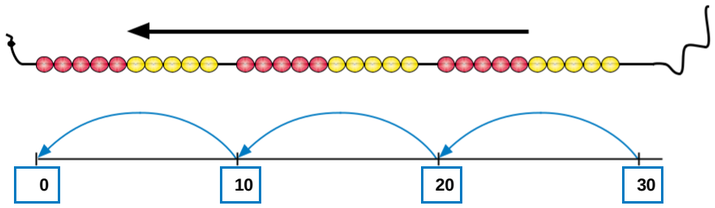
Division

EYFS

Solve problems by sharing objects in a practical or role play context.

EYFS

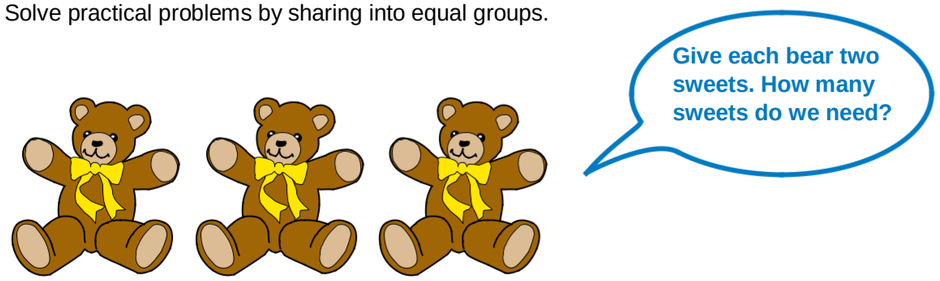
Count back in 10s.



NB: *Counting on is a powerful tool for mental calculation but does not lead onto written calculation for division.*

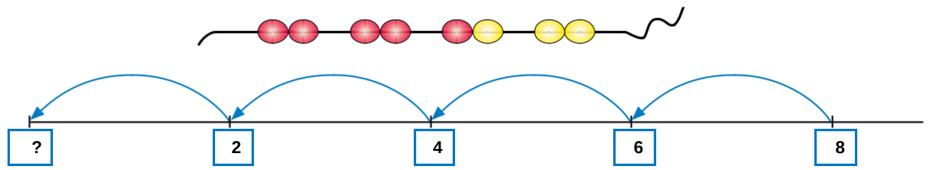
EYFS

Solve practical problems by sharing into equal groups.

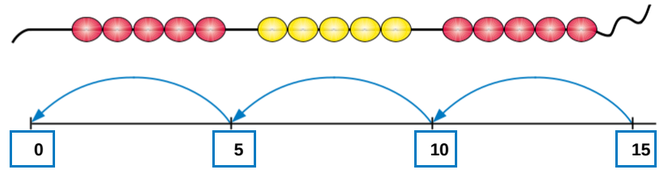


Year 1

Count back in 2s.

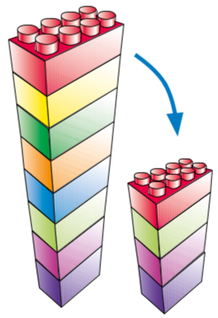


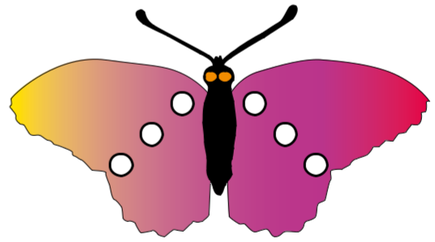
Count back in 5s.

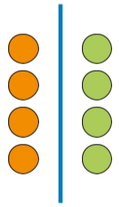


Year 1

Know halves to 20.







Half of 8 is 4

8÷2=4

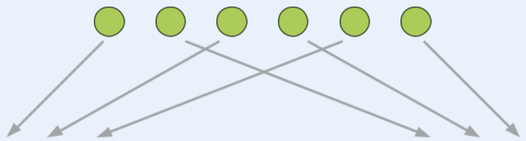
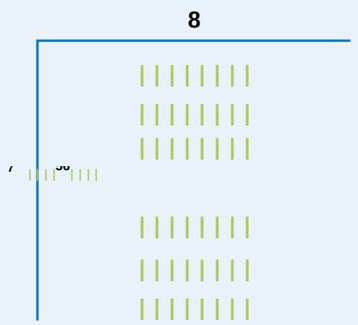
Half of 6 is 3

6÷2=3

CONCRETE PICTORIAL ABSTRACT 2+1=3

Written methods for division

These notes show the stages in building up to long division in 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

Mental calculation strategies	Stage 1: Mental division using partitioning	CONCRETE APPROACHES
Derive quickly: • Doubles of numbers to at least 5 and corresponding halves • Doubles of numbers to 10 and multiples of 10 and corresponding halves • Doubles of whole numbers to 20, multiples of 5 to 100 and 50 to 500 and corresponding halves **Know by heart:** • Multiplication facts for 2, 5 and 10 times tables and corresponding divisions • Multiplication facts for 2, 3, 4, 5 and 10 times tables and corresponding divisions To divide by 10/100, shift the digits one/two places to the left	Sharing **6 ÷ 2**  Grouping **6 ÷ 2**  Repeated subtraction: **6÷2=6–2–2–2=0** Partitioning **84 ÷ 7** might be: **84** **70 + 14** **÷ 7** **10 + 2= 12** Grid method **x 7 70 14** **x 10 2** **7 70 14** **10+2=12**	 **56**  **56 counters in groups of 7**  **8** **7 5 6**

CONCRETE PICTORIAL ABSTRACT 2+1=3

Mental calculation strategies

Derive quickly:

- Doubles of numbers to at least 5 and corresponding halves
- Doubles of numbers to 10 and multiples of 10 and corresponding halves
- Doubles of whole numbers to 20, multiples of 5 to 100 and 50 to 500 and corresponding halves

Know by heart:

- Multiplication facts for 2, 5 and 10 times tables and corresponding divisions
- Multiplication facts for 2, 3, 4, 5 and 10 times tables and corresponding divisions

To divide by 10/100, shift the digits one/two places to the left

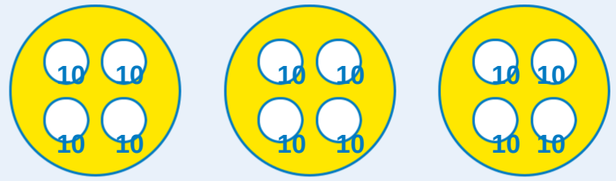
Stage 2: 'Expanded' method for HTU ÷ U

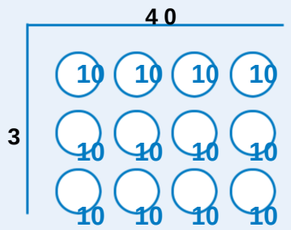
120÷3

```
    4 0
  ┌────
3 │ 1 2 0
```

CONCRETE APPROACHES

120÷3





Mental calculation strategies

Derive quickly:

- Doubles of whole numbers to 50, multiples of 10 to 500 and 100 to 5000 and corresponding halves
- **Know by heart:**
- All multiplication facts to 10 x 10 and corresponding divisions
- To divide by 4, halve and halve again (and for finding ¼)
- To divide by 5, divide by ten and double (and for finding 1/5)
- To divide by 20, divide by 10 and halve

'Expanded' method for long division

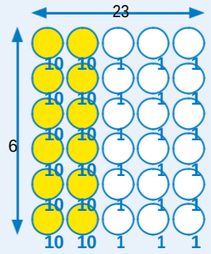
How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

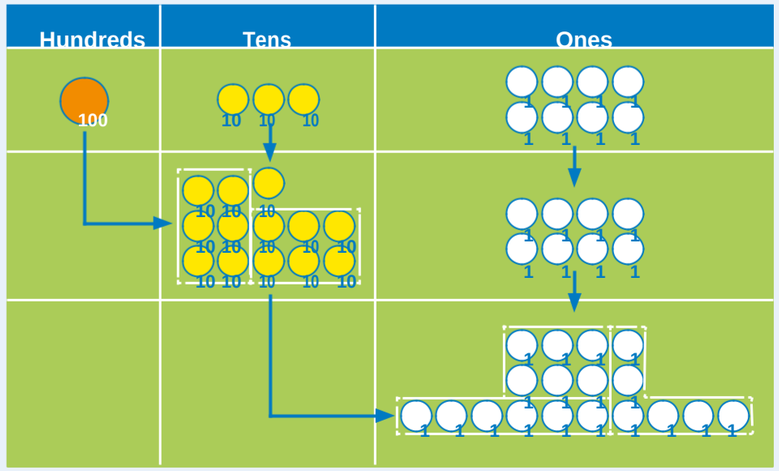
```
    23   R8
  ┌────
24│ 560
  – 480   24 x 20
    80
  – 72    24 x 3
    8
```

24 x 20

CONCRETE APPROACHES

```
    2 3
  ┌────
6 │ 0 1 1
  │ 1 3 8
```





Informal to standard written calculations

year	Addition	Subtraction	Multiplication	Division
3	**TU + TU developing to HTU + TU or HTU + HTU** 1. Use of number lines to count on 2. Horizontal expanded method, using partitioning 3. Vertical expanded method adding most (or least) significant digit first	**TU – TU, developing to HTU – TU or HTU – HTU** 1. Use of number line to count up 2. Use of number line to take too much and add back 3. Use of partitioned vertical form (expanded form) 4. Decomposition using expanded form	• Repeated addition • Describing an array • Concrete written • Scaling	• Grouping • Sharing • Remainders
4	**HTU + TU then HTU + HTU** 1. Vertical expanded method adding most significant digit first 2. Vertical expanded method adding least significant digit first 3. Leading to regrouping below the line 4. Calculations extended to include addition of two or more 3-digit sums of money	**HTU – TU then HTU – HTU** 1. Decomposition using expanded form 2. Decomposition using compact form 3. Calculations extended to include the difference between two 3-digit sums of money	**TU x U** 1. Grid method (TU x U) 2. Standard expanded short multiplication (TU x U) 3. Compact short multiplication (TU x U)	**TU÷U** 1. TU ÷ U – using chunking
5	**HTU + HTU then ThHTU + ThHTU** 1. Vertical expanded method adding least significant digit first 2. Compact written method regrouping below the line 3. Calculations extended to include addition of two or more decimal fractions, with up to three digits and the same number of decimal places, in vertical format	**HTU – HTU, then ThHTU – ThHTU** 1. Decomposition using expanded form 2. Decomposition using compact form 3. Calculations extended to include subtraction of decimals, with up to 3 digits & the same number of decimal places, in expanded format leading to vertical format	**HTU x U and TU x TU** 1. Grid method (HTU x U & TU x TU) 2. Standard expanded short multiplication (HTU x U) 3. Compact short multiplication (HTU x U) 4. Long multiplication (TU x TU) 5. Calculations extended to include multiplying decimal fractions with one decimal place by a single digit	**HTU÷U** 1. HTU ÷ U – using chunking 2. HTU ÷ U – efficient chunking
6	**ThHTU + ThHTU and then any number of digits** 1. Compact written method regrouping below the line 2. Calculations extended to include addition of two or more decimal fractions with up to four digits & either one or two decimal places	**ThHTU – ThHTU and then any number of digits** 1. Decomposition using compact form 2. Calculations extended to include subtraction of two or more decimal fractions with up to 3 digits & either one or two decimal places in vertical format	**ThHTU x U and HTU x TU** 1. Grid method (ThHTU x U & HTU x TU) 2. Standard expanded short multiplication (ThHTU x U) 3. Compact short multiplication (ThHTU x U) 4. Long multiplication (HTU x TU) 5. Calculations extended to include multiplying decimal fractions with two decimal places by a single digit	**HTU ÷ TU** 1. HTU ÷ TU – using chunking 2. HTU ÷ TU – efficient chunking 3. HTU ÷ TU – efficient standard method 4. Extend to decimal fractions with up to two decimal places

Summary

1. Children should always estimate first.
2. Always check the answer, preferably using a different method eg. the inverse operation.
3. Always decide first whether a mental method is appropriate.
4. Pay attention to language - refer to the actual value of digits.
5. Children who make persistent mistakes should return to the method that they can use accurately until ready to move on.
6. Children need to know number and multiplication facts by heart.
7. Discuss errors and diagnose problem and then work through problem – do not simply re-teach the method.
8. When revising or extending to harder numbers, refer back to expanded methods. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.